Algebraic Formula Sheet

Arithmetic Operations

$$ac + bc = c(a+b)$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{c} - \frac{c}{d} = \frac{ad-bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c}$$

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c,\ a \neq 0$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Properties of Exponents

$$x^n x^m = x^{n+m}$$

$$x^0 = 1,\ x \neq 0$$

$$(x^n)^m = x^{nm}$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$(xy)^n = x^n y^n$$

$$\frac{1}{x^{-n}} = x^n$$

$$x^{\frac{n}{m}} = \left(x^{\frac{1}{m}}\right)^n = \left(x^n\right)^{\frac{1}{m}}$$

$$\frac{x^n}{x^m} = x^{n-m}$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$$

$$x^{-n} = \frac{1}{x^n}$$

Properties of Radicals

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$

$$\sqrt[n]{x^n} = x,\ \text{if } n \text{ is odd}$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$$

$$\sqrt[n]{x^n} = |x|,\ \text{if } n \text{ is even}$$

Properties of Inequalities

If $a < b$ then $a + c < b + c$ and $a - c < b - c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|x| \geq 0$$

$$|-x| = |x|$$

$$|xy| = |x||y|$$

$$\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$$

$|x+y| \leq |x| + |y|$ **Triangle Inequality**

$|x-y| \geq \Big||x| - |y|\Big|$ **Reverse Triangle Inequality**

Distance Formula

Given two points, $P_A = (x_1, y_1)$ and $P_B = (x_2, y_2)$, the distance between the two can be found by:

$$d(P_A, P_B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Number Classifications

Natural Numbers : $\mathbb{N}$={1, 2, 3, 4, 5, . . .}

Whole Numbers : {0, 1, 2, 3, 4, 5, . . .}

Integers : $\mathbb{Z}$={... ,-3, -2, -1, 0, 1, 2, 3, .. .}

Rationals : $\mathbb{Q}$={All numbers that can be written as a fraction with an integer numerator and a nonzero integer denominator, $\frac{a}{b}$}

Irrationals : {All numbers that cannot be expressed as the ratio of two integers, for example $\sqrt{5}$, $\sqrt{27}$, and π}

Real Numbers : $\mathbb{R}$={All numbers that are either a rational or an irrational number}

Logarithms and Log Properties

Definition

$y = \log_b x$ **is equivalent to** $x = b^y$

Example

$\log_2 16 = 4$ **because** $2^4 = 16$

Special Logarithms

$\ln x = \log_e x$ **natural log**
where e=**2.718281828...**

$\log x = \log_{10} x$ **common log**

Logarithm Properties

$\log_b b = 1$ $\qquad$ $\log_b 1 = 0$

$\log_b b^x = x$ $\qquad$ $b^{\log_b x} = x$

$\ln e^x = x$ $\qquad$ $e^{\ln x} = x$

$\log_b \left(x^k\right) = k \log_b x$

$\log_b (xy) = \log_b x + \log_b y$

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

Factoring

$xa + xb = x(a + b)$

$x^2 - y^2 = (x + y)(x - y)$

$x^2 + 2xy + y^2 = (x + y)^2$

$x^2 - 2xy + y^2 = (x - y)^2$

$x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3$

$x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3$

$x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$

$x^3 - y^3 = (x - y)\left(x^2 + xy + y^2\right)$

$x^{2n} - y^{2n} = (x^n - y^n)(x^n + y^n)$

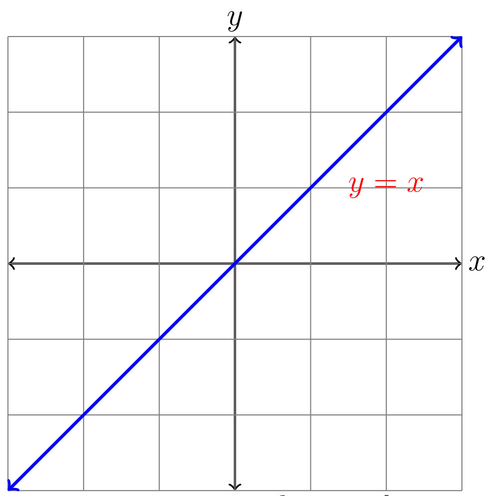
If n is odd then,

$x^n - y^n = (x - y)\left(x^{n-1} + x^{n-2}y + ... + y^{n-1}\right)$

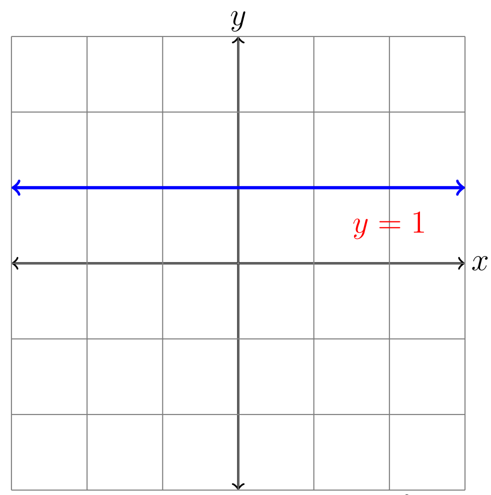
$x^n + y^n = (x + y)\left(x^{n-1} - x^{n-2}y + x^{n-3}y^2 ... - y^{n-1}\right)$

Linear Functions and Formulas

Examples of Linear Functions



linear function



constant function

Constant Function

This graph is a horizontal line passing through the points (x, c) with slope $m = 0$:

$$y = c \quad or \quad f(x) = c$$

Linear Function/Slope-intercept form

This graph is a line with slope m and $y - intercept(0, b)$:

$$y = mx + b \quad or \quad f(x) = mx + b$$

Slope (*a.k.a Rate of Change*)

The slope m of the line passing through the points (x_1, y_1) and (x_2, y_2) is :

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}$$

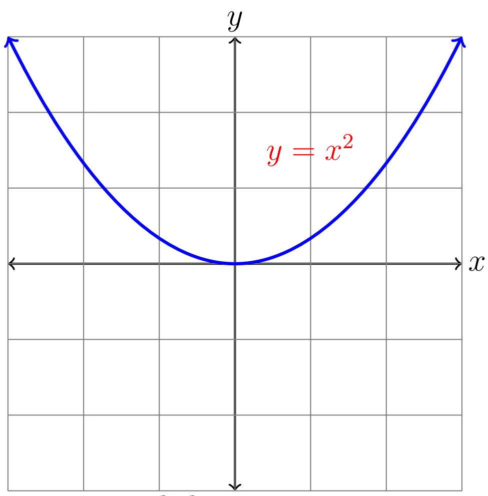
Point-Slope form

The equation of the line passing through the point (x_1, y_1) with slope m is :

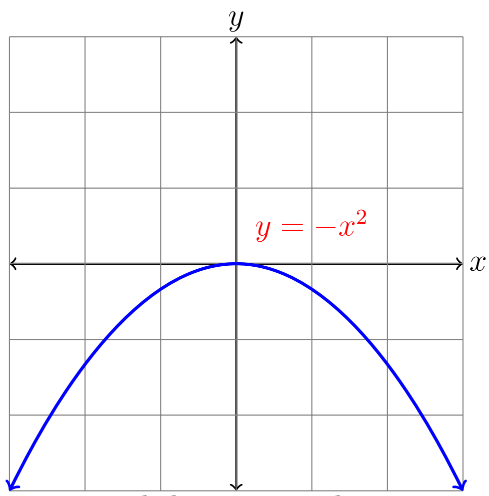
$$y = m(x - x_1) + y_1$$

Quadratic Functions and Formulas

Examples of Quadratic Functions



parabola opening up



parabola opening down

Forms of Quadratic Functions

Standard Form

$$y = ax^2 + bx + c$$
or
$$f(x) = ax^2 + bx + c$$

This graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right).$$

Vertex Form

$$y = a(x - h)^2 + k$$
or
$$f(x) = a(x - h)^2 + k$$

This graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k).

Quadratics and Solving for x

Quadratic Formula
To solve $ax^2 + bx + c = 0, \ \ a \neq 0$,
use :
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

The Discriminant
The discriminant is the part of the quadratic equation under the radical, $b^2 - 4ac$. We use the discriminant to determine the number of real solutions of $ax^2 + bx + c = 0$ as such :

1. If $b^2 - 4ac > 0$, there are two real solutions.
2. If $b^2 - 4ac = 0$, there is one real solution.
3. If $b^2 - 4ac < 0$, there are no real solutions.

Square Root Property

Let k be a nonnegative number. Then the solutions to the equation

$$x^2 = k$$

are given by $x = \pm\sqrt{k}$.

Other Useful Formulas

Compound Interest

$\mathbf{A} = P\left(1 + \dfrac{r}{n}\right)^{nt}$

where:
P= principal of P dollars
r= Interest rate (expressed in decimal form)
n= number of times compounded per year
t= time

Continuously Compounded Interest

$\mathbf{A} = Pe^{rt}$

where:
P= principal of P dollars
r= Interest rate (expressed in decimal form)
t= time

Circle

$(x-h)^2 + (y-k)^2 = r^2$

This graph is a circle with radius r and center (h,k).

Ellipse

$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

This graph is an ellipse with center (h,k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

This graph is a hyperbola that opens left and right, has center (h,k), vertices $(h \pm a, k)$; foci $(h \pm c, k)$, where c comes from $c^2 = a^2 + b^2$ and asymptotes that pass through the center $y = \pm\dfrac{b}{a}(x-h) + k$.

$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

This graph is a hyperbola that opens up and down, has center (h,k), vertices $(h, k \pm a)$; foci $(h, k \pm c)$, where c comes from $c^2 = a^2 + b^2$ and asymptotes that pass through the center $y = \pm\dfrac{a}{b}(x-h) + k$.

Pythagorean Theorem

A triangle with legs a and b and hypotenuse c is a right triangle if and only if

$a^2 + b^2 = c^2$